General Approach for Homogenisation of Multi-turn Windings in Three-dimensional Finite Element Models

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1 Introduction

Multi-turn windings in electromagnetic devices may be subjected to considerable skin and proximity effect. Taking into account this effect accurately in a FE model of a complete device remains a challenge as the brute-force approach, consisting in discretising each separate turn of the winding, is in most cases extremely expensive in terms of memory requirements and computation time. Homogenisation methods are therefore indispensable. In the frequency domain, they usually amount to the use of complex and frequency-dependent reluctivity and resistance values, the expression of which is obtained analytically, Moreau et al [1], or using an elementary FE model, Podoltsev and Lebedev [2].
Gyselinck and Dular [3] have proposed a more general approach, in which conductors of arbitrary cross-section and packing can be considered. In the present paper, the incorporation of the homogenisation approach in a 3D FE model is presented. By way of validation the method is applied to the 3D model of an axisymmetric multi-turn inductor, for which a brute-force 2D FE model provides an accurate reference solution.

2 Characterisation of the winding

A complete eddy-current effect characterisation of the winding (i.e. conductor cross-section, packing type, and fill factor) can be carried out by means of a representative 2D FE model consisting of a central cell and one or more layers of cells around it [3]. Each elementary cell comprises the cross-section of one conductor and the surrounding isolation. In Figs. 1 and 2, such a representative FE model is shown for the conductor that will be considered in this paper, viz a round conductor with square packing and fill factor λ equal to 0.65. In [3] rectangular conductors and hexagonal packing are considered as well.
The normalised or *reduced frequency* X is defined as:

$$X \;=\; \frac{r}{\delta} \;=\; \sqrt{f}\cdot r\sqrt{\pi\sigma\mu_0}\,, \qquad (1)$$

where r is the (equivalent) radius of the conductors, δ the penetration depth at frequency f or pulsation $\omega = 2\pi f$, σ the conductivity of the conductors and $\mu_0 = 4\pi 10^{-7}$ H/m their permeability.
The elementary FE model is excited with a unit current of frequency X in the conductors and zero net flux (skin effect) or with a unit horizontal (or vertical) induction and zero net current (proximity effect). By integrating j^2/σ, with j the current density, over the central conductor and

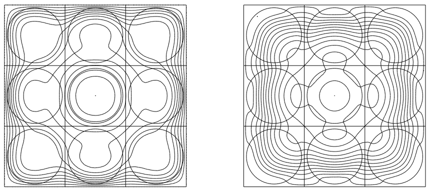

Fig. 1 Elementary FE model with skin effect flux – components in phase and in quadrature with the imposed unit current (right and left resp.) with $X = 2$

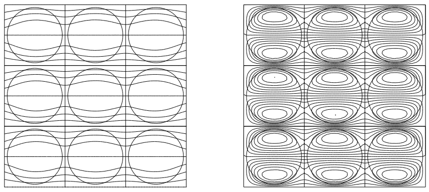

Fig. 2 Elementary FE model with proximity effect flux – components in phase and in quadrature with the imposed unit horizontal flux (right and left resp.) with $X = 2$

$\omega b^2/\mu_0$, with b the flux density, over the central cell, the active and reactive power consumption of the winding cells is quantified. This results in two dimensionless and frequency-dependent skin-effect coefficients $p_I(X)$ and $q_I(X)$, as well as in two (or four, in the more general anisotropic case) proximity effect coefficients $p_B(X)$ and $q_B(X)$. They are 1 or close to 1 for X sufficiently small, as shown in Figure 3.

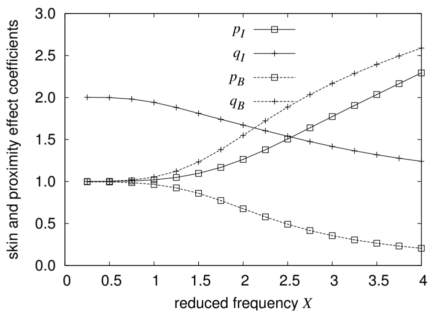

Fig. 3 Skin and proximity effect coefficients versus frequency X for round conductors with square packing ($\lambda = 0.65$)

3 Homogenisation in 3D FE model

The proximity effect in the winding is accounted for by adopting in the homogenised winding area (copper plus isolation) the complex reluctivity $\boldsymbol{\nu}_{prox}(X) = \nu_0 \left(q_{B,x}(X) + i\, p_{B,x}(X)\, \frac{\lambda X^2}{2} \right)$ for the two perpendicu-

lar directions in the plane of the conductor section.
As for the skin effect, the DC resistance R_{DC} in the electrical circuit equations has to be replaced by the impedance $\boldsymbol{Z}_{skin}(X) = R_{DC}\left(p_I(X) + i\, q_I(X)\,\frac{X^2}{4}\right)$.
The homogenisation method is applied to an axisymmetric 120-turn inductor. The round conductor ($1\,\mathrm{mm}^2$ section) and square packing ($\lambda = 0.65$) of the previous section, together with the skin and proximity effect coefficients, are adopted. The magnetic core ($\mu_r = 1000$) is either closed (”no airgap”) or has a central 3 mm airgap.
In a 3D FE model of the inductor, see Fig. 4, it is out of the question to model each turn separately. The induction produced by a uniform current density in the homogenised winding is calculated by means of a 3D magnetic vector potential formulation [4].

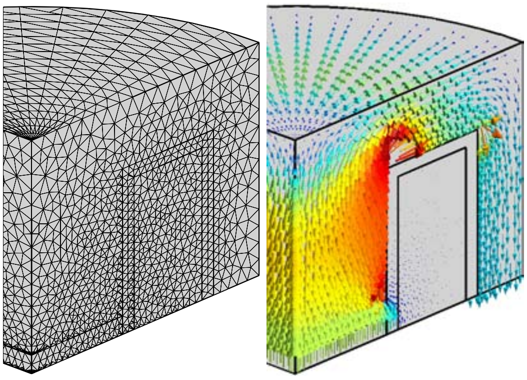

Fig. 4 3D FE model of axisymmetric inductor (one eighth of geometry) with homogenised coil : mesh and induction ($X = 2$ with airgap)

In a 2D FE model, it is workable to discretise each separate turn finely, see Figure 5. A 1 A current is imposed in each turn. Some flux patterns are shown in Figure 6.

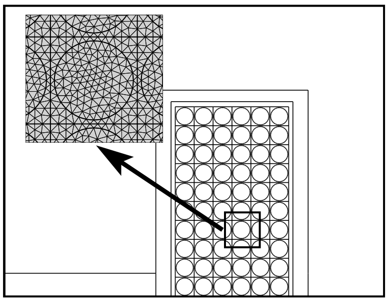

Fig. 5 2D FE model of axisymmetric 120-turn inductor (half height) – detail of the mesh

The resistance $R(X)$ and inductance $L(X)$ of the winding are calculated with the 2D and 3D FE model. An excellent agreement is observed in Figures 7 and 8. The eddy current losses very much increase with frequency, mainly due to proximity effect, whereas the inductance is only very slightly affected.

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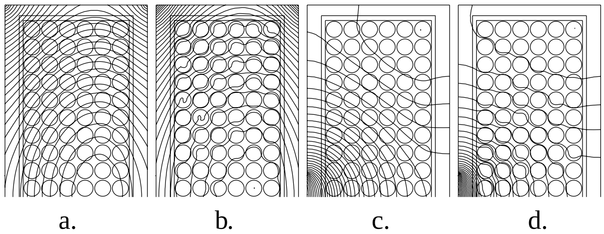

Fig. 6 Flux lines in winding window for reduced frequency $X = 0.25$ (a and c) and $X = 2$ (b and d), without airgap (a and b) and with airgap (c and d)

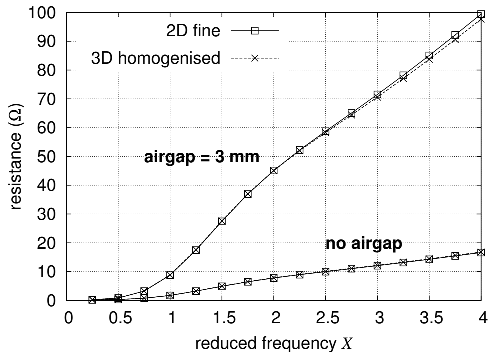

Fig. 7 Resistance versus reduced frequency X

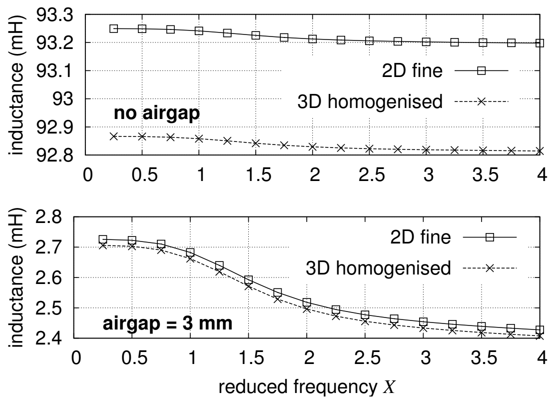

Fig. 8 Inductance versus reduced frequency X

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